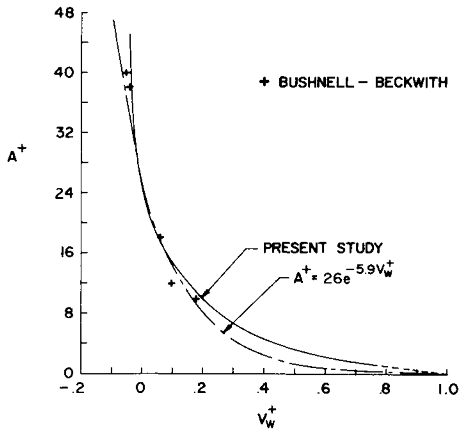

Fig. 1 Variation of A^+ with v_w^+.

For equilibrium boundary layers we can write

$$(R_{\delta*}/\delta^+) = (1+\Pi)/\kappa \tag{13}$$

With that relation and with the definition of c, the right-hand side of (12) can be written as

$$(1/\kappa)\ln \kappa R_{\delta}*/(1+\Pi)+c_o+(2z/\bar{v}_w)[(1+10.805\bar{v}_w z^2)^{1/2}-1]-10.805+2\Pi/\kappa \tag{14}$$

Equation (12) with its right-hand side given by (14) can now be calculated for given values of v_w, $R_{\delta*}$ and R_θ to find z (or c_f). Knowing c_f, we can obtain the solution of (8), namely u^+, for various values of A^+ and we can determine u_p^+. Away from the wall but in the law of the wall region, u_p^+-distribution will be of the form

$$u_p^+ = (1/\kappa)\ln y^+ + c \tag{15}$$

with c given by (11) provided that we have the "correct" values of A^+ for given values of v_w^+. Thus when a value of A^+ is found that satisfies Eq. (15), the solution of (8) also satisfies the law of the wall portion of (10).

Figure 1 shows the variation of A^+ with v_w^+ obtained by this curve-fitting procedure for the experimental data of Simpson et al.[5] As expected, the damping parameter A^+ decreases with blowing and it increases with suction. For blowing, the curve-fitting procedure can be made for values of $A^+ \geqq 0$ since for $A^+ = 0$, the sublayer region and the buffer region disappear. For suction with increasing v_w^+, the flow may become laminarized thus restricting the upper values of A^+.

Figure 1 also shows the values of A^+ given by Bushnell and Beckwith[6] and by Cebeci.[7] Of these, Bushnell and Beckwith's values were obtained from the experimental data of Simpson et al.[5] Cebeci's values were obtained by a different extension of the Van Driest model[1] to boundary layers with mass transfer. The extension was made by interpreting the phase velocity of the oscillations in Stokes flow to be the friction velocity at the edge of the sublayer rather than its wall value which is the case for a flat-plate only. For an incompressible, zero pressure gradient flow with mass transfer, the damping parameter is given by[7]

$$A^+ = 26\exp(-5.9v_w^+) \tag{16}$$

The comparison of the three predicted variations of A^+ and v_w^+, obtained by three different approaches, shows good agreement with each other. It further indicates that methods that use an eddy viscosity formulation such as the one given by (1) and (3) with A^+ given by (16) or a mixing length formulation such as the one used by Bushnell and Beckwith,[6] are in good agreement with Coles's expression[4] for the range $-0.05 \leqq v_w^+ < 1.0$.

References

[1] Van Driest, E. R., "On Turbulent Flow Near a Wall," *Journal of the Aeronautical Sciences*, Vol. 23, No. 11, Nov. 1956, pp. 1007–1011.

[2] Klebanoff, P. S., "Characteristics of Turbulence in a Boundary Layer with Zero Pressure Gradient," TN 3178, July 1954, NACA.

[3] Cebeci, T., "Kinematic Eddy Viscosity at Low Reynolds Numbers," *AIAA Journal*, Vol. 11, No. 1, Jan. 1973, pp. 102–104.

[4] Coles, D., "A Survey of Data for Turbulent Boundary Layers with Mass Transfer," *Conference Proceedings, AGARD*, London, England, Sept. 1971.

[5] Simpson, R. L., Kays, W. M., and Moffat, R. J., "The Turbulent Boundary Layer on a Porous Plate: An Experimental Study of the Fluid Dynamics with Injection and Suction," Rept. HMT-2, 1967, Stanford Univ., Stanford, Calif.

[6] Bushnell, D. M. and Beckwith, I. R., "Calculation of Non-equilibrium Hypersonic Turbulent Boundary Layers and Comparisons with Experimental Data," *AIAA Journal*, Vol. 8, Aug. 1970, pp. 1462–1469.

[7] Cebeci, T., "The Behavior of Turbulent Flow Near a Porous Wall with Pressure Gradient," *AIAA Journal*, Vol. 8, No. 12, Dec. 1970, pp. 2152–2156.

Contribution in Unsteady Flow of Power-Law Fluids

SREEDHAN ROY*
University of Aston in Birmingham, Birmingham, England

Nomenclature

C_f = coefficient of skin friction, Eq. (9)
$F(\eta)$ = dimensionless velocity
$G(\eta)$ = dimensionless velocity, Eq. (3)
N = parameter in constitutive equation for power-law fluids
ε = parameter, Eq. (5)
η = similarity variable

RECENTLY, Chen and Wollersheim[1] have presented solutions for power-law fluids ($0.25 \leqq N \leqq 3.0$) for both impulsively started plate and flow cases. We shall show that the appropriate similarity equation for the velocity distribution admits of a perturbation treatment based on the assumption that the fluid is slightly non-Newtonian. The same technique has been applied in the case of Blasius flow in power-law fluids by Roy.[2]

The equation to be solved is[1]

$$F'' = 2\eta(-F')^{2-N} \tag{1}$$

subject to the boundary conditions

$$F(0) = 1, \qquad F(\infty) = 0 \tag{2}$$

Substituting

$$F = 1 - G \tag{3}$$

in Eqs. (1) and (2) we have

$$G'' + 2\eta(G')^{2-N} = 0, \quad G(0) = 0, \quad G(\infty) = 1 \tag{4}$$

Let us now assume that the fluid is slightly non-Newtonian and that

$$N = 1+\varepsilon \quad (\varepsilon \text{ being small}), \quad G = G_0 + \varepsilon G_1 + \varepsilon^2 G_2 \tag{5}$$

Equations (4) and (5) now lead to

$$G_0'' + 2\eta G_0' = 0, \quad G_1'' + 2\eta G_1' - 2\eta G_0' \log G_0' = 0 \tag{6a}$$

$$G_2'' + 2\eta G_2' - 2\eta\{G_1'(1+\log G_0') - \tfrac{1}{2}G_0'(\log G_0')^2\} = 0 \tag{6b}$$

$$G_0(0) = 0, \quad G_0(\infty) = 1, \quad G_1(0) = G_1(\infty) = G_2(0) = G_2(\infty) = 0 \tag{6c}$$

Equations (6) have been solved numerically on an electronic computer. The solutions lead to the results

Received June 27, 1972.
Index categories: Boundary Layers and Convective Heat Transfer—Laminar; Nonsteady Aerodynamics.

* Research Fellow, Department of Mechanical Engineering.

Table 1 Numerical values of skin-friction parameters

N	$G'(0)$ Ref. 1	Ref. 3	Roy	$C_f(N)$ Ref. 1	Ref. 3	Roy
0.25	0.6663	...	0.7144	0.9925	...	1.0099
0.50	0.8694	0.871	0.8852	0.8145	0.816	0.8219
0.75	1.0213	...	1.0232	0.6718	...	0.6727
1.00	1.128	1.128	1.1284	0.564	0.564	0.5642
1.25	1.2024	...	1.2007	0.4823	...	0.4815
1.50	1.2531	...	1.2402	0.4187	...	0.4123
1.75	1.2875	...	1.2469	0.3683	...	0.3483
2.00	1.3104	...	1.2207	0.3276	...	0.2843

$$G_0'(0) = 1.128379, \quad G_1'(0) = 0.354998, \quad G_2'(0) = -0.262704 \tag{7}$$

Thus,

$$G'(0) = 1.128379 + 0.354998\varepsilon - 0.262704\varepsilon^2 \tag{8}$$

Another result of importance is the drag-coefficient C_f which is given by (Ref. 1)

$$C_f(N) = \frac{[G'(0)]^N}{[2N(N+1)]^{N/(N+1)}} \tag{9}$$

In Table 1 we compare the results given by present analysis with those of Ref. 1 and Wells.[3] It may be observed that the perturbation results are almost exact in the region $0.5 \leqq N \leqq 1.5$.

References

[1] Chen, T. Y. W. and Wollersheim, D. E., "Solution of Unsteady Flow of Power Law Fluids," *AIAA Journal*, Vol. 10, No. 5, May 1972, pp. 689–691.

[2] Roy, S., "On Blasius Flow in Non-Newtonian Fluids," *AIChE Journal*, Vol. 18, No. 3, May 1972, p. 666.

[3] Wells, C. S., Jr., "Unsteady Boundary-Layer Flow of a Non-Newtonian Fluid on a Flat Plate," *AIAA Journal*, Vol. 2, No. 5, May 1964, pp. 951–952.

Reference Temperature Method for Predicting Turbulent Compressible Skin-Friction Coefficient

FRANK L. CLARK* AND THEODORE R. CREEL JR.*
NASA Langley Research Center, Hampton, Va.

Nomenclature

C_f = local skin-friction coefficient
M_∞ = Mach number
T = temperature, °R
R_θ = momentum thickness Reynolds number
θ = momentum thickness
μ = viscosity, lb/ft sec
$(\bar{\ })$ = incompressible flow

Subscripts

aw = adiabatic temperature
c = calculated
D = measured values
w = wall conditions
∞ = freestream conditions

Superscript

$(\ )'$ = parameters evaluated at reference temperature conditions

Received July 5, 1972; revision received September 15, 1972.

Index category: Boundary Layers and Convective Heat Transfer—Turbulent.

* Aerospace Engineer, Aerodynamics & Heat Transfer Section, Hypersonic Vehicles Division.

IN the past decade, numerous reference-temperature methods have been proposed as a means of computing skin-friction coefficient in a compressible turbulent boundary layer.[1-3] Unfortunately at the time these methods were developed, only a limited amount of directly measured compressible skin friction existed; most of the data were obtained on flat plates (zero longitudinal pressure gradient) at low Mach numbers and low heat-transfer rates. In addition, a large quantity of the skin-friction coefficient data were obtained by sloping the calculated results from profiles of total pressure and temperature surveys of the boundary layer nearest the wall. This approach can lead to erroneous values of skin friction because of the distortion of the boundary layer due to the interaction of the probe and the model surface[6] and local low Reynolds number effects.

The present Note presents a new reference-temperature relationship to improve the accuracy of predicting the value of skin-friction coefficient in a compressible turbulent boundary layer for a wide range of Mach numbers and wall temperature ratios in the presence of a slightly favorable pressure gradient (nozzle wall). Only experiments in which the skin friction was directly measured (free-floating element) and momentum thickness Reynolds numbers calculated from the results of total pressure and temperature surveys were used in the analysis. The data used cover a range of Mach numbers from approximately 2–20 and wall temperature ratios (T_w/T_{aw}) from 0.1 to 1.0.[4-10]

Results

The following relationships between compressible and incompressible flows were used

$$\bar{C}_f/C_f = T'/T_\infty, \qquad \bar{R}_\theta/R_\theta = \mu_\infty/\mu' \tag{1}$$

where the value of μ for air is represented by Keyes' viscosity equation[12]

$$\mu = 7.47\times10^{-7}(T)^{1/2}[1+(220/T)\times10^{-9/T}]^{-1} \tag{2}$$

and the value μ for nitrogen[13] is given by

$$\mu = 6.8873\times10^{-7}(T)^{1/2}(1+180/T)^{-1} \tag{3}$$

The values of turbulent incompressible local skin-friction coefficient were computed from the Kármán-Schoenherr equation.[11]

Values of skin-friction coefficient and momentum thickness Reynolds number are used to solve Eq. (1) iteratively to obtain values of T'/T_∞. Since the Eckert method[1] adequately predicts the value of compressible skin-friction coefficient for the condition of wall temperature equal to recovery temperature, it is also used in the present approach; the results of the calculations are

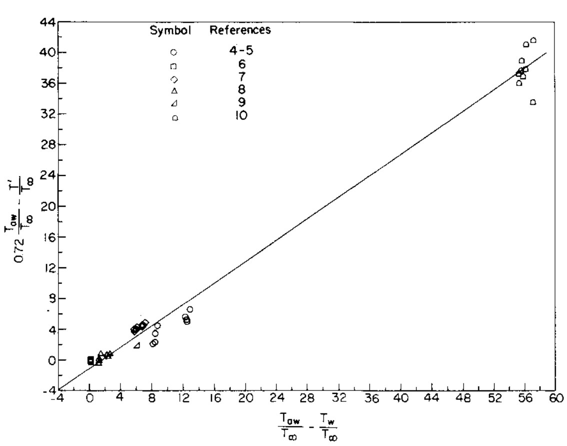

Fig. 1 Calculation of reference temperature for turbulent flow for slightly favorable pressure gradient.